ON THE SOLVABILITY OF A NONLINEAR BOUNDARY CONTROL PROBLEM FOR OSCILLATION PROCESSES DESCRIBED BY FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

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In the paper, we investigated the solvability of nonlinear optimization problem, where required to minimize the functional

$$J[u(t)] = \int_Q \{[V(T,x) - \xi_1(x)]^2 + [V_t(T,x) - \xi_2(x)]^2\}dx + \beta \int_0^T p^2[t,u(t)]dt, \quad \beta > 0,$$

on the set of solutions of the boundary value problem

$$V_{tt} - AV = \lambda \int_0^T K(t,\tau)V(\tau,x)d\tau + g(t,x), \quad x \in Q \subset R^n, \quad 0 < t \leq T,$$

$$V(0,x) = \psi_1(x), \quad V_t(0,x) = \psi_1(x), \quad 0 < x < 1,$$

$$\Gamma V(t,x) \equiv \sum_{i,j=1}^n a_{ij}(x)V_{x_j}(t,x)cos(\delta,x_i) + a(x)V(t,x) = b(t,x)p[t,u(t)], \quad x \in \gamma, \quad 0 < t \leq T,$$

where Q is a region of the space R^n, with γ boundary, $\xi_1(x) \in H(Q)$, $\xi_2(x) \in H(Q)$, $\psi_1(x) \in H_1(Q)$, $\psi_2(x) \in H(Q)$, $K(t,\tau) \in H(D)$, $D = \{0 \leq t \leq T, \quad 0 \leq \tau \leq T\}$, $g(t,x) \in H(Q_T)$, $Q_T = Q \times (0,T)$, $b(t,x) \in H(Q_\gamma)$, $Q_\gamma = \gamma \times (0,T)$ are known functions; $AV(t,x) = \sum_{i,j=1}^n \left(a_{ij}(x)V_{x_j}(t,x)\right)_{x_i} - c(x)V(t,x)$ is an elliptic operator; $a(x) \geq 0$, $c(x) \geq 0$ are known measurable functions; $p[t,u(t)] \in H(0,T)$ is a given function, which depends nonlinearly from the control function $u(t) \in H(0,T)$ nonlinearly and satisfies the condition $p_u[t,u(t)] \neq 0$; λ is a parameter, T is a fixed moment of time; $H(X)$ is a Hilbert space of functions defined on the set X; $H_1(Q)$ is a first order Sobolev space.

- The algorithm for constructing generalized solution of the boundary value problem was developed and the convergence of approximate solutions was proved;
- The optimality conditions of control was found by the maximum principle for systems with distributed parameters in the form of equality and differential inequality. The algorithm was developed for constructing solution of nonlinear integral equation of the optimal control;
- Solution of optimization problem was obtained in the form of the triple $\left(u^0(t), \quad V^0(t,x), \quad J[u^0(t)]\right)$, where $u^0(t)$ is the optimal control, $V^0(t,x)$ is the optimal process, $J[u^0(t)]$ is the functionals minimum value.