An analysis of the Blackstock–Crighton equation in L_p-spaces

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We are going to address the question of optimal regularity and long-time behavior of solutions for the Blackstock–Crighton equation given in terms of the acoustic velocity potential u,

$$(a\Delta - \partial_t)(u_{tt} - b\Delta u_t - c^2\Delta u) = (k(u_t)^2 + |\nabla u|^2)_{tt},$$

which models the propagation of finite-amplitude sound in thermoviscous fluids. Here, a is the thermal diffusivity, b is the acoustic diffusivity, c is the speed of sound and k denotes the parameter of nonlinearity. We consider the Blackstock–Crighton equation together with non-homogeneous Dirichlet boundary conditions.

We show that, for small initial and boundary data, there exists a unique global solution which depends continuously on the data and converges to zero at an exponential rate as time tends to infinity. The regularity of the initial and boundary data is necessary and sufficient for the regularity of the solution. Our techniques are based on maximal L_p-regularity for parabolic problems and the implicit function theorem.

Furthermore, there will be a short outlook on how to treat the case of non-homogeneous Neumann boundary conditions which are relevant for applications such as modeling boundary excitation of high intensity focused ultrasound for medical purposes.