Sparsity constrained image restoration: an approach using the Newton Projection method

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This work is concerned with the general problem of image restoration under sparsity constraints formulated as

$$\min_{\mathbf{x}} \quad \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + \lambda\|\mathbf{x}\|_1 \tag{1}$$

where $\mathbf{A}$ is a $m \times n$ real matrix (usually $m \leq n$), $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{b} \in \mathbb{R}^m$ and λ is a positive parameter. In some image restoration applications, $\mathbf{A} = \mathbf{K}\mathbf{W}$ where $\mathbf{K} \in \mathbb{R}^{m \times n}$ is a discrete convolution operator and $\mathbf{W} \in \mathbb{R}^{n \times n}$ is a transformation matrix. The data $\mathbf{b}$ is the measurements vector which is assumed to be affected by Gaussian white noise. Recently, a modified Newton projection (MNP) method has been proposed [Landi G., *JMIV*, 51:195–208, 2015] for the *analysis* formulation of problem (1), where $\mathbf{W}$ is the identity matrix and $\mathbf{x}$ represents the image itself. In this work, the MNP method is extended to the *synthesis* formulation of problem (1), where $\mathbf{W}$ contains a wavelet basis or a redundant dictionary and the variable $\mathbf{x}$ represents the image coefficients which are known to be sparse. In the proposed approach, problem (1) is firstly formulated as a nonnegatively constrained quadratic programming problem by splitting the variable $\mathbf{x}$ into its positive and negative parts. Then, the quadratic program is solved by a special purpose MNP method where a fair regularized approximation to the Hessian matrix is employed so that the only operations required for the search direction computation are matrix-vector products involving $\mathbf{K}$, $\mathbf{W}$ and their transpose. As a result, the search direction can be efficiently obtained. Convergence of the proposed MNP method is proved under standard assumptions. Even if the size of the problem is doubled, the low computational cost per iteration and less iterative steps make MNP quite efficient. The performance of MNP is evaluated on several image restoration problems and is compared with that of some state-of-the-art methods. The results of the comparative study show that MNP is competitive and in some cases is outperforming some state-of-the-art methods in terms of computational complexity and achieved accuracy.

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