Threshold strategies in optimal stopping problems with applications to investment timing decisions

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Many problems in the theory of investment under uncertainty ("real options", e.g., an investment timing problem, sequential investment) can be reduced to optimal stopping problems for diffusion processes. The investment decision can be determined as a solution of an optimal stopping problem, which is often generated by threshold strategies. This means that optimal stopping time is specified as the first time when the observed process exceeds a certain threshold (a threshold structure stopping time).

In this paper we consider an optimal stopping problem for one-dimensional regular diffusion process over an infinite time interval. The goal of the paper is to obtain the optimality conditions for threshold strategies in optimal stopping problem.

The paper presents two results. The first one describes necessary and sufficient conditions for optimality over the class of threshold strategies. The second result states the necessary and sufficient conditions for threshold structure of optimal stopping times over the class of all stopping times. In other words we give the complete characterization of optimal stopping problems, for which the solution has a threshold structure.

The results are used to justify the investment threshold decisions in two problems. The first problem concerns the known investment timing problem. The second one is finding an optimal time for abandonment of oil extraction under falling oil prices.