Optimal Control of Gradient Systems

Tuğba Güney* & Bülent Karasözen & Murat Uzunca
Institute of Applied Mathematics & Department of Mathematics,
Middle East Technical University, Ankara-Turkey
*guney.tugba@metu.edu.tr

In this talk we consider optimal control of gradient systems with the Ginzburg-Landau free energy

$$\frac{\partial u}{\partial t} = -\frac{\delta F}{\delta u}, \quad F(u) = \int_\Omega \left(\frac{1}{\epsilon^2} E(u,v) - \frac{1}{2}(\nabla u)^2 \right) \mathrm{d}\Omega, \quad \Omega \in \mathbb{R}^d, \ d = 1, 2.$$

and double well potential $E(u,v) = \frac{1}{4}u^4 - \frac{1}{2}u^2 + vu$, $\epsilon > 0$. Examples of gradients systems are Schlögl equation[2] arising in chemical waves and Allen-Cahn equation [1], basic model for diffuse interface problems. Gradient systems are characterized by energy decreasing property $F(u(t)) \leq F(u(s)), \ s > t$. Numerical integrators that preserve the energy decreasing property in the discrete setting are called energy or gradient stable. It is known that the implicit Euler method is first order unconditionally energy stable method. The second order unconditionally energy stable method is average vector field (AVF) integrator [4]. We discretize the gradients systems by discontinuous Galerkin method [6] in space and by the implicit Euler method and AVF integrator in time. We solve optimal control problems for the Schlögl equation and Allen-Cahn equation with travelling and spiraling waves using sparse [3] and H_1 regularized [5] controls.

References

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