

# Optimal Control of Piecewise-Smooth System of Differential Delay Equations with Application in Immunology

Irina Bolodurina and Ulia Ivanova

The main purpose of the study is to provide a broad overview of important ideas in analysis and search of optimal control nonsmooth system of differential equations with delay, presented in the form of

$$\dot{x}(t) = \begin{cases} f_1(t, x(t-\tau), x(t), u(t)), & S(t, x) < 0, \\ f_2(t, x(t-\tau), x(t), u(t)), & S(t, x) \geq 0, \end{cases} \quad (1)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$  - vectors of the state and control respectively,  $\tau$  - constant lag,  $S(t, x)$  - scalar function continuously differentiable by the set of arguments and determining the violation of smoothness of the right part of the system (1). The main focus of this work is the problem of necessary optimality conditions and their application to constructions numerical methods of search for the solution to practical tasks in the field of immunology, described by a system of differential equations of form (1).

The research examined the different immunological problems of optimal control in modeling the process of antigens reproduction and neutralization, proliferation and differentiation of plasmacells and antibodies. The above tasks differ from each other in various features, such as the dimensionality of phase area, the types of nonlinearities, the restrictions, multiextremality, types of the gaps in the system, as well as the presence of lags in the phase variables etc. For the problems considered specialized methods have been developed for the search of optimal control, a variety of necessary optimality conditions, the algorithms and the software have been obtained allow to find the optimum control and give it a meaningful interpretation.

**Keywords:** optimal control, differential equations with a piecewise-smooth right-hand side and by the delay, mathematical modeling