

# Multimaterial Topology Optimization of Variational Inequalities

Andrzej Myśliński<sup>1</sup>

<sup>1</sup> Systems Research Institute, Warsaw, Poland, [myslinsk@ibspan.waw.pl](mailto:myslinsk@ibspan.waw.pl)

## Abstract

The paper is concerned with the structural topology optimization of systems governed by the variational inequalities. The class of such systems includes among others contact phenomenon between the surfaces of the elastic bodies described by the elliptic variational inequalities. This optimization problem consists in finding such topology of the domain occupied by the body and/or the shape of its boundary that the normal contact stress along the boundary of the body is minimized. In literature this problem usually is considered as two-phase material optimization problem. In recent years multiple phases topology optimization problems have become subject of the growing interest. The use of multiple number of phases during design of engineering structures opens a new opportunities in the design of smart and advanced structures.

The paper deals with the analysis and numerical solution of the topology optimization problem of system governed by the variational inequalities. The domain occupied by the body is assumed to consist from several elastic materials rather than two materials. Material density function is a variable subject to optimization. The regularization of the objective functional by the multiphase volume constrained Ginzburg-Landau energy functional is used. The derivative formula of the cost functional with respect to the material density function is calculated and is employed to formulate a necessary optimality condition for the topology optimization problem. This necessary optimality condition takes the form of the generalized Allen -Cahn equation.. The derivative of the cost functional appears in the right hand side of these equation. Moreover the cost functional derivative is employed to calculate a descent direction in the numerical algorithm. Finite element method is used as the approximation method. Implementation details are introduced. Numerical examples are provided and discussed.

**Key words:** multimaterial topology optimization; variational inequalities; multiphase Allen-Cahn problem