Partial Differential-Algebraic Equations (PDAes) and Optimal Control

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In recent years, the idea and notions of so-called Partial Differential-Algebraic Equations (PDAEs) have received increased attention. We consider optimal control problems constrained by such equations, which we interpret as abstract DAEs on Banach spaces. This translates to coupled systems of each a parabolic- and a (quasi-)stationary elliptic equation for two unknown functions.
In DAE theory, the concept of an Index plays a crucial role in determining the properties of the system. We impose a general condition corresponding to equations of Index 1, namely, that the partial derivative of the elliptic equation with respect to the searched-for function there is continuously invertible, which has multiple consequences. These are then used to establish general conditions for existence of solutions to the whole system via e.g. maximal parabolic regularity. Here, the underlying domains and associated function spaces may be of very general nature.
From there, we switch to optimal control theory, where we complement the PDAE system with an appropriate objective functional and possibly state constraints for the function associated to the parabolic equation. Due to the linear nature of the systems involved in the usual necessary first-order optimality conditions, the Index 1 assumption implies more useful properties, since it in fact means that the linearized elliptic equation is uniquely solvable and allows to again rely on maximal parabolic regularity (possibly non-autonomous) for further investigations.
We then apply the general theory to a real-world problem under appropriate assumptions.

(this is joint work with Stefan Ulbrich)