Optimal Control of convective FitzHugh-Nagumo model

Bülent Karasözen * & Murat Uzunca & Tuğba Güney
Institute of Applied Mathematics & Department of Mathematics,
Middle East Technical University, Ankara-Turkey
*bulent@metu.edu.tr

In this talk we investigate optimal control of wave propagation in excitable media described by two-dimensional FitzHugh-Nagumo model. The model consists of two coupled reaction-diffusion-convection equations describing the flow in blood coagulation and in bioreactors [2]

$$\begin{aligned}\frac{\partial u}{\partial t} &= d_u \Delta u - V(y)\frac{\partial u}{\partial x} + \alpha u(u-\beta)(1-u) - v,\\ \frac{\partial v}{\partial t} &= d_v \Delta v - V(y)\frac{\partial v}{\partial x} + \epsilon(\gamma u - v),\end{aligned}$$

where u and v are activator and inhibitor, respectively and $V(y)$ is the velocity profile. The flow plays an important role by the regularization of the excitation threshold and wave propagation. The plane waves occurring in two-dimensional media are controlled in different ways. We solve optimal control problem using the all-at-once approach and sparse [1] and H_1 regularized [3] controls. For space discretization we use the symmetric interior penalty discontinuous Galerkin method [4] and for time discretization the implicit Euler method.

References

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