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Inverse Problems in Applied Sciences

A weighted wavelet method for region of interest tomography

Esther Klann

Technische Universität Berlin

klann@math.tu-berlin.de

Abstract: We consider the *interior problem* or *region of interest (ROI) tomography*; tomographic data are given only over lines meeting a region of interest Ω inside an object, and the goal is to image that region, i.e., to compute f on Ω from $Rf = z$ from given (possibly noisy) ROI data z^δ. Already the problem of reconstructing a function from its full tomographic data is ill-posed and regularization methods have to be applied. We consider the ROI problem for piecewise constant functions that can be written as the finite linear combination of characteristic functions: $f(x) = \sum_{n=1}^{N} a_n \chi_{\Omega_n}(x)$.
We use a weighted wavelet reconstruction scheme which leads to the minimization of the functional

$$\|Rf - z^\delta\|^2_{L_2(S^1 \times \mathbb{R})} + \alpha \|Ff\|^p_{p,\omega}$$

where

$$\|Ff\|^p_{p,\omega} = \sum_{j,k} \omega_{jk} |c_{jk}|^p$$

is a weighted ℓ_p-norm with strictly positive weights $0 < C \leq \omega_{jk}$ and the operator F maps f to its Fourier coefficients with respect to the Haar wavelet basis. For $p < 2$, such an approach is known to promote sparsity; and for $p > 1$ the functional is strictly convex and has a unique minimizer. The novelty of the proposed method is that the weights depend on the relative location of wavelets to the ROI. We classify the wavelet basis functions as follows: (1) basis functions with support either containing the region of interest (approximation, coarse wavelets), or being contained in the region of interest (detail, fine wavelet); (2) basis functions with support that does not overlap the region of interest (details, fine wavelets and coarser wavelets away from the ROI); (3) basis functions with support that overlaps the region of interest without belonging to the first type.
We demonstrate numerically the performance of the method for different phantoms.