27th IFIP TC7 Conference 2015 on System Modelling and Optimization

Inverse Problems in Applied Sciences

(p,q)-Laplace Operator for Image Enhancement

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Abstract: A well known inverse problem in image processing is to denoise images. In recent years, the p-parabolic equation associated with the $p-$Laplace operator

$$\Delta_p u = \text{div}(|\nabla u|^{p-2}\nabla u) = |\nabla u|^{p-1}\Delta_1 u + (p-1)|\nabla u|^{p-4}\Delta_\infty u, \quad p \geq 1$$

has been successfully used to enhance images: $p = 2$ is the linear Gaussian filter, $p = 1$ is the method of total variation and $p = 0$ is the so called balanced forward backward evolution. An important feature in any evolution process is preservation of certain geometrical properties of the underlying image, like edges and corners. To this end, Perona and Malik (P-M) proposed in early 1990's to use the weight $g(|\nabla u|) = \frac{1}{1+|\nabla u|^2/k}$ instead of $|\nabla u|^{p-2}$ where the weight should be small for large $|\nabla u|$, which is a peculiar character of edges. To understand why the method of P-M works, we rewrite

$$\text{div}(g(|\nabla u|)\nabla u) = g(|\nabla u|)\Delta_1 u + \phi(|\nabla u|)\Delta_\infty u, \quad \phi(s) = (sg(s))'.$$

Thus, the evolution diffuses differently in the direction ∇u and $\nabla^\perp u$ due to different weights in Δ_1 and Δ_∞. Since $\phi(s)$ becomes negative for large s, the evolution will be of backward diffusion effect near edges. This backward evolution will cause a problem for the well posedness of the model and could also lead to staircasing problem. In this work, we introduce a new evolution equation

$$u_t - \Delta_{(p,q)} u = 0, \qquad u(0) = u_0, \qquad \partial_n u = 0,$$

where the (p,q)-**Laplace operator** $\Delta_{(p,q)}$ is defined as

$$\Delta_{(p,q)} u = |\nabla u|^q \Delta_1 u + (p-1)|\nabla u|^{p-4}\Delta_\infty u, \quad p \in (1,2), \; q > 0,$$

and u_0 is the damaged image. Simulations shows that our method yields better results compared to the P-M method.