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Optimal control and Hamilton-Jacobi-Bellman equations: Numerical methods and Applications

Dynamic Programming using Radial Basis Functions

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Abstract:
For many optimal control problems, solutions can elegantly be characterized and computed by *dynamic programming*, i.e. by solving a fixed point equation, the *Bellman equation*, for the optimal value function of the problem. For every point in the state space of the underlying problem, this function yields the optimal cost associated to this initial condition. At the same time, the value function allows to construct an optimal controller in feedback form, enabling a robust stabilization of a possibly unstable nominal system. This controller not only yields optimal cost trajectories, but also a maximal domain of stability for the closed loop system.
In this talk, we consider a discrete time optimal control problem (which can be obtained from some HJB equation by a discretization in time with fixed time step) and show that using radial basis functions in combination with a moving least squares type approximation (aka *Shepard's method*) one obtains a simple, yet general scheme for the numerical solution of the problem. After a brief review on radial basis functions, we derive the Shepard-RBF discretization of the optimality principle, show convergence of the value iteration and present several numerical experiments as well as a simple yet general MATLAB code.