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Transport metrics in image and shape processing

Learning Graph Metrics for Optimal Transport

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Abstract: Optimal transport can be used to lift metrics from a base space to the set of probability measures thereon. The usefulness of this metric for various problems in image analysis and machine learning has been widely accepted. In most applications the base space metric is assumed to be given a priori or to be chosen by the user. But clearly a manually designed metric is not necessarily optimal for a given task and in more abstract settings it may even be completely unclear what a suitable metric should look like. The problem to learn the base space metric for optimal transport from data has first been studied in [2]. The major challenges are the computational complexity of solving many transport problems, the parametrization of the feasible set of metrics, in particular ensuring the triangle inequality, and the optimization w.r.t. the metric.
We discuss a new approach to the metric learning problem for optimal transport on graphs. The metric is parametrized by local edge lengths, thus exploiting graph sparsity and avoiding the need to enforce the triangle inequality. A variant of entropic smoothing [1] allows application of simple local Bregman projection algorithms for the transport problems and simplifies the optimization w.r.t. the metric.
This is joint work with Gabriel Peyré and Marco Cuturi.

References

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[2] M. Cuturi and D. Avis. Ground metric learning. *Journal of Machine Learning Research*, 15:533–564, 2014.