27th IFIP TC7 Conference 2015 on System Modelling and Optimization

Modeling, Estimation, and Control Related Issues in Physical and Biological Applications

Optimal Perturbations to Enhance Information Content in Data Sets for a Heat Equation

D. Rubio, N. Saintier, M.I. Troparevsky, H.T.Banks

Universidad Nacional de San Martin, Universidad de Buenos Aires, Universidad Nacional de San Martin, NC State University

drubio@unsam.edu.ar

Abstract:
In this work we consider the parametric heat equation

$$\begin{aligned} \frac{\partial u}{\partial t} &= c\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, 0 < t < T, \\ u(0,t) &= kb(t), \qquad 0 < t < T, \\ \frac{\partial u}{\partial x}(1,t) &= h(t)(u(1,t) - u_0), \qquad 0 < t < T, \\ u(x,0) &= u_0(x), \qquad 0 < x < 1. \end{aligned}$$

The diffusivity coefficient c is to be estimated based on a set of measured data $u(1, \tau_k; b)$, $0 < \tau_1 < \tau_2, ..., \tau_k < T, \quad b \in B$, where $B = \left\{ b = \sum_{i=1}^{H} b_i \aleph_{[t_{i-1}, t_i]},\ 0 < t_0 < ... < t_H < T \right\}$. The perturbation functions b describe whether the input is on or off in each of the intervals $[t_{i-1}, t_i], \quad i = 1, ..., H$.
In order to maximize information content in the data set, optimal design is performed on both, the family B and the observation instants $\tau_1, ..., \tau_k$, following [1].
Statistical methods will be used to evaluate the accuracy of the estimation.

[1] K. Adoteye, H.T. Banks, K.B. Flores, *Optimal Design of Non-equilibrium Experiments for Genetic Network Interrogation*, Applied Mathematics Letters, **40** (2015), 84-89.