

Double Convergence of a Family of Discrete Distributed Elliptic Optimal Control Problems

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Abstract: We consider a bounded domain Ω in \mathbb{R}^n whose regular boundary $\partial\Omega$ consists of the union of two disjoint portions Γ_1 and Γ_2 with $meas(\Gamma_1) > 0$. The convergence of a family of continuous distributed mixed elliptic optimal control problems P_α , governed by elliptic variational equalities (EVEs), when the parameter α of the family (the heat transfer coefficient on the portion of the boundary Γ_1) goes to infinity was studied in Gariboldi-Tarzia, Appl. Math. Optim., 47 (2003), 213-230. It has been proved that the optimal control, and their corresponding system and adjoint system states are strongly convergent, in adequate functional spaces, to the optimal control, and the system and adjoint system states of another distributed mixed elliptic optimal control problem P governed also by another EVE with a different boundary condition on the boundary Γ_1 .

We consider the discrete approximations $P_{h\alpha}$ and P_h of the optimal control problems P_α and P respectively, for each $h > 0$ and for each $\alpha > 0$, through the finite element method with Lagrange's triangles of type 1 with parameter h (the longest side of the triangles). We also discrete the EVEs which define the system and their adjoint system states, and the corresponding cost functional of the distributed optimal control problems P_α and P .

The goal of this paper is to study the double convergence of this family of discrete distributed mixed elliptic optimal control problems $P_{h\alpha}$ when the parameter $\alpha \rightarrow +\infty$ and the parameter $h \rightarrow 0$. We prove the convergence of the discrete optimal controls, the discrete system and adjoint system states of the family $P_{h\alpha}$ to the corresponding to the discrete distributed mixed optimal control problem P_h when $\alpha \rightarrow +\infty$, for each $h > 0$, in adequate functional spaces. We study the convergence of the discrete optimal control problems $P_{h\alpha}$ and P_h when $h \rightarrow 0$ and we obtain a commutative diagram which relates the continuous and discrete distributed mixed elliptic optimal control problems $P_{h\alpha}$, P_h , P_α and P by taking the limits $h \rightarrow 0$ and $\alpha \rightarrow +\infty$ respectively. We also study the double convergence of $P_{h\alpha}$ to P when $(h, \alpha) \rightarrow (0, +\infty)$ which represents the diagonal convergence in the previous commutative diagram.