

A Criterion for Robust Stability with Respect to Parametric Uncertainties Modeled by Multiplicative white Noise with Unknown Intensity, with Applications to Stability of Neural Networks

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Abstract: In this talk we deal with the problem of exponential stability in mean square (ESMS) of linear stochastic systems of the form:

$$dx(t) = A(\eta_t)x(t)dt + \sum_{k=1}^r \mu_k b_k(\eta_t) c_k^T(\eta_t) x(t) dw_k(t), \quad t \geq 0 \quad (1)$$

where $\{w_k(t)\}_{t \geq 0}, 1 \leq k \leq r$ are independent standard 1-dimensional Wiener processes and $\{\eta_t\}_{t \geq 0}$ is a standard right continuous Markov process taking values in the finite set $\{1, 2, \dots, N\}$. In (1) $A(i) \in \mathbf{R}^{n \times n}, b_k(i), c_k(i) \in \mathbf{R}^{n \times 1}$ are known matrices while, the scalars μ_k are unknown and they represent the intensity of the noises modeled by the Wiener processes. The system (1) may be regarded as a perturbation of the nominal system

$$\dot{x}(t) = A(\eta_t)x(t). \quad (2)$$

Assuming that the nominal system (2) is ESMS we want to provide an estimation of the domain of variation of the unknown parameters μ_k (named region of stability) such that the property of ESMS be preserved by the corresponding perturbed system of type (1). The obtained criterion for describing the stability region in the space of parameters $\mu = (\mu_1, \mu_2, \dots, \mu_r)$ is expressed by the condition that the spectral radius of a matrix suitable associated to system (1) to be less than 1. The theoretical developments are illustrated by a numerical example in which a Hopfield network with output feedback is considered which nonlinear activation function is assumed to satisfy a sector-type condition. The slope uncertainty of this sector is approximated by a Markov jump parameter. Then the stability robustness of the stochastic neural network is analyzed using the main results of this paper.