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Regularized Optimal Design Problem for a Viscoelastic Plate Vibrating against a Rigid Obstacle

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Abstract: Shape design optimization problems belong to frequently solved problems with many engineering applications. We deal here with an optimal design problem for a viscoelastic anisotropic plate vibrating against a rigid obstacle. A variable thickness of a plate plays the role of a control variable. We assume the middle surface Ω of the plate, the constant material density $\rho > 0$, a variable thickness $x \mapsto e(x),\ x = (x_1, x_2)$ and a function $\Phi : \bar{\Omega} \to \mathbb{R}$ expressing the form of the obstacle. Due to the variable thickness the relations for the movement of the plate acting under the perpendicular force $F(t, x)$ and the unknown contact force G have the form

$$\rho e(x)w_{tt} + [e^3(x)(A_{ijkl}w_{t,x_ix_j} + B_{ijkl}w_{x_ix_j})]_{x_kx_\ell} = F + G,\ 0 \le G \perp u - \tfrac{1}{2}e - \Phi \ge 0 \text{ in } (0, T] \times \Omega.$$

In order to derive not only the existence of an optimal variable thickness e but also the necessary optimality conditions we formulate a regularized problem using a nondecreasing function $g_\delta \in C^2(\mathbb{R}) \ge 0$ of the variable ω vanishing for $\omega \le 0$, equaled to ω for $\omega \ge \delta > 0$ and fulfilling $\max_{\omega\in[0,\delta]} |g_\delta(\omega)| \le M\delta$. Assuming the plate clamped on the boundary $\partial\Omega$ we solve the hyperbolic state initial-boundary value problem for a deflection $u \equiv u(e)$

$$e(x)u_{tt} + [e^3(x)(a_{ijkl}u_{t,x_ix_j} + b_{ijkl}u_{x_ix_j})]_{x_kx_\ell} + \frac{1}{\delta}g_\delta(u - \tfrac{1}{2}(e(x) - \Phi(x))) = f(t, x),$$

$$u(t, \xi) = \frac{\partial u}{\partial \vec{n}}(t, \xi) = 0,\ (t, \xi) \in (0, T] \times \partial\Omega;\ u(0, x) = u_0(x),\ u_t(0, x) = v_0(x),\ x \in \Omega,$$

$$e \in E_{ad} = \Big\{e \in H^2(\Omega) :\ 0 < e_{\min} \le e(x) \le e_{\max}\ \forall x \in \bar{\Omega},\ \|e\|_{H^2(\Omega)} \le \hat{e}\Big\}$$

together with

Optimal Design Problem $\mathcal{P}$: To find a control (thickness) $e_* \in E_{ad}$ such that

$$J(u(e_*), e_*) \le J(u(e), e)\ \forall e \in E_{ad}.$$

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