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Adaptivity and memory-reduced adjoints for optimization problems with parabolic PDE-constraints

Adaptive inexact Newton methods and adaptive regularization and space–time discretization for unsteady nonlinear problems

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Abstract: We first describe the adaptive inexact Newton method of [1]. Herein, to solve a nonlinear algebraic system arising from a numerical discretization of a steady nonlinear partial differential equation, we consider an iterative linearization (for example the Newton or the fixed-point ones), and, on its each step, an iterative algebraic solver (for example the conjugate gradients or GMRes). We derive adaptive stopping criteria for both these iterative solvers. Our criteria are based on an a posteriori error estimate which distinguishes the different error components, namely the discretization error, the linearization error, and the algebraic error. We stop the iterations whenever the corresponding error does no longer affect the overall error significantly. Our estimates hinge on equilibrated flux reconstructions. They yield a guaranteed upper bound on the overall error measured by the dual norm of the residual augmented by a jump nonconformity term. Our estimates are valid at each step of the nonlinear and linear solvers. Importantly, we prove their (local) efficiency and robustness with respect to the size of the nonlinearity.

We then turn to a model nonlinear unsteady (degenerate) problem, the Stefan solidification, and consider its conforming spatial and backward Euler temporal discretizations. Following [2], we show how to derive estimators yielding a guaranteed and fully computable upper bound on the dual norm of the residual, as well as on the $L^2(L^2)$ error of the temperature and the $L^2(H^{-1})$ error of the enthalpy. Extending the above steady case developments, we distinguish all the space, time, regularization, linearization, and algebraic error components. An adaptive algorithm is in particular proposed, which ensures computational savings through the online choice of a sufficient regularization parameter, a stopping criterion for the linearization iterations, local space mesh refinement, time step adjustment, and equilibration of the spatial and temporal errors. A theoretical proof of the efficiency of our estimate, as well as illustrative numerical experiments, are presented.

References

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[2] D. Di Pietro, M. Vohralík, and S. Yousef, *Adaptive regularization, linearization, and discretization and a posteriori error control for the two-phase Stefan problem*, Math. Comp. **84** (2015), 153–186.