27th IFIP TC7 Conference 2015 on System Modelling and Optimization

Inverse problems for elliptic PDEs, analysis and applications

Partially overdetermined problem for Laplace equation: taking the most from measured data while smoothing the boundary

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Abstract: Motivated by the general physical setup when geometry of the domain is approximately known such that only part of the boundary admits small perturbations (typically due to peculiar sensor positions, as best illustrated on Figure), we cast the following two-dimensional field recovery problem:

$$\begin{cases} \Delta u = 0 & \text{in } \Omega, \\ u = u_0, \ \partial_n u = \omega_0 & \text{on } \Gamma, \end{cases}$$

where $\Omega \subset \mathbb{R}^2$ is simply-connected, $\partial \Omega \in Lip$, $\Gamma \subset \partial \Omega$, and $u_0 \in W^{1/2,2}_{\mathbb{R}}(\Gamma)$, $\omega_0 \in L^2_{\mathbb{R}}(\Gamma)$ are real-valued. Such ill-posed formulation is converted into analytic function recovery problem with pointwise constraints, and we further obtain its explicit solution in terms of series expansion over the regularization parameter.

This is a joint work with Juliette Leblond and Laurent Baratchart.

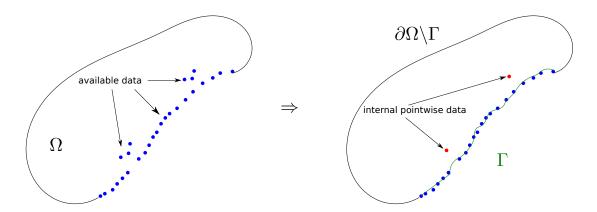


Figure 1: Sample geometry of the problem and its modification