27th IFIP TC7 Conference 2015 on System Modelling and Optimization

Wellposedness, control, and observability theories for partial differential equations

Wellposedness and Stability Solutions to Quasilinear Systems Arising in Hyperbolic-Parabolic Dynamics-Applications to Free Boundary Problems and Fluid Structure Interactions

Irena Lasiecka

University of Memphis
Memphis, TN, USA

lasiecka@memphis.edu

Abstract: Quasilinear system consisting of a 3-D Navier Stokes equation coupled with 3D wave equation will be considered. The coupling occurs via boundary traces on the interface between the fluid and the solid. This PDE arises in the context of modeling free boundary problems, in particular fluid-structure interactions (e.g., submerged submarine or blood flowing through the aorta). The main topic discussed is existence and uniqueness of appropriately defined solutions. The primary, well-recognized, difficulty is due to a mismatch between parabolic and hyperbolic regularity. It turns out that new developments in linear hyperbolic and parabolic theories such as

(i) *maximal L_p parabolic regularity* for the Stokes operator;

(ii) *sharp boundary trace regularity* in hyperbolic dynamics provide fundamental bearing and tools on solvability of this quasilinear system.

Maximal parabolic regularity allows to determine the pressure from finite energy solutions of the Stokes problem, while sharp trace regularity allows to propagate the boundary traces through the interface without a loss of regularity (whereas, classical trace theorems in L_p scales result in a loss of $1/p$ of a derivative when restricting to the boundary). These two ingredients prevent a typical "loss" of derivatives and allow for the algebraic and topological closure of a suitably constructed fixed point.
Both the concepts (i) and (ii) will be explained within the linear framework. Standard approaches to quasilinear theories will be outlined. It will be shown why these fail when dealing with mixed parabolic and hyperbolic dynamics. Finally the strategy leading to a construction of quasilinear theories based on the ingredients (i) and (ii) above will be presented. The final results include solution to a long-standing open problem on global solvability of quasilinear hyperbolic-parabolic system.