27th IFIP TC7 Conference 2015 on System Modelling and Optimization

Wellposedness, control, and observability theories for partial differential equations

Boundary Control of a Schrödinger Equation with a Point-mass Interface

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Abstract: First I'll describe several results and some open problems regarding regularity and boundary controllability of elastic systems involving an interior point mass. Some new problems of this type arise when considering thermoelastic systems or Schrodinger's equation. In particular, I'll focus on the case of Schrodinger equation on the domain $(-1,0)\cup(0,1)$ with a transmission condition at $x=0$ that describes a δ-function potential well:

$$\begin{cases} u_t + iu_{xx} = 0, & x\in(-1,0)\cup(0,1),\ t>0 \\ u(0^-,t) = u(0+,t) & t>0 \\ \frac{d}{dt}u(0,t) + i[u_x(0^+,t) - u_x(0^-,t)] = 0 & t>0 \\ u(-1,t) = 0, & t>0 \end{cases}$$

with either Dirichlet control : $u(1,t) = f(t)$, or Neumann control: $u_x(1,t) = f(t)$. In the Neumann case, we find results analogous to known results for the wave equation, in which exact controllability holds on a space with differing regularities on each side of the interface. This is not the case however in the case of Dirichlet control, where the controllability space is the same on each side. Our approach is based on the moment method and non-harmonic Fourier series.