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Model reduction and uncertainty quantification for parameter estimation

Sparsity formulations in OED

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Abstract: In this talk we consider Optimal Design of Experiments (OED) for a parameter-dependent model given by a linear, elliptic equation. As a simple example we consider

$$-\triangle y + qy = f \quad on\ \Omega$$
$$y = 0 \quad on\ \partial\Omega$$

with a positive constant q and an $L^2(\Omega)$ function f. However, we will also discuss more general situations with parameters in $\mathbb{R}^n$.

To fit the model to the reality, the true parameter q^* has to be estimated indirectly, e.g. from pointwise measurements of the state. Due to measurement errors such an estimate can be seen as a realization of a random variable and the size of its confidence regions is an indicator for the quality of this estimate. An important task in OED is to find an optimal design ω (optimal locations for the sensors and an optimal number of measurements) to obtain estimates which are more reliable. This is usually done by minimizing a suitable function on the eigenvalues of the linearized Fisher-information matrix $C(\omega)$ over a set of possible experimental designs.
In our approach, possible designs ω are modelled by the space of finite Radon-measures, which allows to consider pointwise measurements (Dirac-Deltas) at arbitrary spatial points as well as continous measurement methods. The costs of the measurements are represented by the total-variation norm. This leads to the optimal control problem

$$\min_{\omega \in M(\Omega)} tr(C(\omega)^{-1}) + \beta \|\omega\|_{M(\Omega)}$$
$$s.t \quad \omega \geq 0$$

We investigate existence of solutions and derive optimality conditions for this problem, leading to a description of the structure of optimal designs. Furthermore we study the dependence of optimal solutions on β and prove the existence of solutions consisting of finitely many Dirac-Deltas.
Finally, we propose an iterative first-order algorithm which adds/removes Delta peaks to/from the previous iterate in every step.