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Some aspects of Variational Analysis and Applications

A priori estimates of minimizers of an integral functional extending the variational Strong Maximum Principle

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Abstract: We consider the variational problem

$$\min\left\{\int_\Omega [f(\rho_F(\nabla u(x))) + \sigma u(x)]\ \mathrm{d}x : u(\cdot) \in u^0(\cdot) + W_0^{1,1}(\Omega)\right\} \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ is an open bounded connected region; $\rho_F(\cdot)$ is the Minkowski functional of a compact convex set $F \subset \mathbb{R}^n$ with $0 \in \operatorname{int} F$; $f : \mathbb{R}^+ \to \mathbb{R}^+ \cup \{+\infty\}$ is a convex lower semicontinuous function with $f(0) = 0$; $u^0(\cdot) \in W^{1,1}(\Omega)$ and σ is a real parameter.

We give some *a priori* estimates of continuous minimizers in the problem (1) near their extreme as well as nonextreme points, which are reduced to an extended version of the Strong Maximum Principle for "elliptic" variational problems in the case $\sigma = 0$, while, on the other hand, generalize the well-known property of solutions to the Poisson equation. The results are obtained by using the technique of Convex Analysis and special minimizers of (1) introduced earlier by A. Cellina. We observe some "cross effect" in the (local) estimates regarded to the sign of the parameter σ, which disappears as $\sigma \to 0$.