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Some aspects of Variational Analysis and Applications

Strong and weak convexity: application to the gradient projection algorithm

Maxim V. Balashov

MOSCOW INSTITUTE OF PHYSICS AND TECHNOLOGY

balashov73@mail.ru

Abstract: We shall consider the next problems in a real Hilbert space H:

$$\max(\text{or}\min)_{x\in A} f(x), \tag{1}$$

where $f : H \to \mathbb{R}$ is a continuous function, $A \subset H$ is a closed convex bounded subset.

Theorem 1 ([1]). Suppose that a set $A \subset H$ is strongly convex of radius $r > 0$, a function $f : H \to \mathbb{R}$ has the Lipschitz continuous gradient on the set A with constant $C > 0$, $r < \frac{m}{C}$. Then the for any initial point $x_0 \in \partial A$ the iteration process

$$x_{k+1} = \arg\max_{x\in A}(f'(x_k), x), \qquad k = 0, 1, 2, 3, \dots,$$

converges to the (unique) solution $z_0 \in A$ of the problem (1-max case) with the rate of geometric progression with common ratio $q = \frac{rC}{m} < 1$: $\|x_{k+1} - z_0\| \le q\|x_k - z_0\|$, for all k.

Theorem 2. Let $A \subset H$ be a proximally smooth subset with constant of proximal smoothness $R > 0$. Let $f : H \to \mathbb{R}$ be a strongly convex function with constant of strong convexity $\varkappa > 0$. Let $\alpha \in \mathbb{R}$, $\mathbb{L}_f(\alpha) \cap A \ne \emptyset$ and the function f has the Lipshcitz continuous gradient with constant $L > 0$ on the set $\mathbb{L}_f(\alpha)$. (Here $\mathbb{L}_f(\alpha) = \{x \in H \mid f(x) \le \alpha\}$). Put $m = \sup_{x\in\mathbb{L}_f(\alpha)} \|f'(x)\|$. Suppose that $\frac{m}{\varkappa} < R$. Then for any initial point $x_0 \in A \cap \mathbb{L}_f(\alpha)$ the iteration process $x_{k+1} = P_A\left(x_k - tf'(x_k)\right), \quad t = \frac{\varkappa - \frac{m}{R}}{L^2 - \frac{\varkappa m}{R}}$, converges to the unique solution $z_0 \in A$ of the problem (1-min case) with the rate of geometric progression with common ratio

$$q(t) = \frac{R}{R - tm}\sqrt{1 - 2t\varkappa + t^2L^2} \in (0, 1), \quad t = \frac{\varkappa - \frac{m}{R}}{L^2 - \frac{\varkappa m}{R}}. \tag{2}$$

References

[1] M. V. Balashov, Maximization of a function with Lipschitz continuous gradient, Fundam. Prikl. Mat., 18:5 (2013), 17-25 (in Russian).