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Optimality principles, dynamic programming, and applied problems

An effective algorithm to constract optimal strategies in nonlinear two-dimensional differential games[1]

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Abstract: We consider a differential game with dynamics $\dot{x}(t) = a(t, x(t), u(t)) + b(t, x(t), v(t))$ on a fixed time segment $[0, T]$. The state vector $x(t)$ is two-dimensional. The player's controls $u(t)$ and $v(t)$ are subjected to the geometrical constraints $u(t) \in P$, $v(t) \in Q$, where $P \subset \mathbb{R}^p$ and $Q \subset \mathbb{R}^q$ are predefined closed bounded sets. The pursuer (first player) can choose the function $u(t)$ and his aim is to bring the phase vector to the predefined target set $M \subset \mathbb{R}^2$ in the termination time: $x(T) \in M$. The evader (second player) can choose the function $v(t)$ and his aim is opposite: $x(T) \notin M$.

We propose an algorithm for calculation of the player's piecewise-constant strategies. For a sufficiently large natural number K_0, we consider the uniform partition of the time segment $[0, T]$ by time moments $t_k = \tau k$, where $\tau = \frac{T}{K_0}$.

The piecewise-constant strategy of the pursuer u^{str} is determined by a set of functions $u_k : \mathbb{R}^n \to P$, $k = \overline{0, K_0 - 1}$. The admissible realization of the evader's control is a measurable function $v : [0, T] \to Q$. The *motion* for an initial state x_0, an admissible realization of the evader's control v and the pursuer's piecewise-constant strategy u^{str} is an absolutely continuous function $x(t) = x^{\text{mot}}(t, x_0, v, u^{\text{str}})$ such that $x(0) = x_0$ and

$$\dot{x}(t) = a(t, x(t), u_k(x(t_k))) + b(t, x(t), v(t)) \quad \forall t \in [t_k, t_{k+1}], \quad k \in \overline{0, K_0 - 1}.$$

The evader's piecewise-constant strategy v^{str}, the admissible realization of the pursuer's control u and the motion $x^{\text{mot}}(t, x_0, u, v^{\text{str}})$ are defined analogically. We say that the piecewise-constant strategy u^{str} *guarantees ε-pursuit* starting from initial state x_0 if the distance between the vector $x^{\text{mot}}(T, x_0, v, u^{\text{str}})$ and the target set M is less than ε for all admissible realization of the evader's control v. The piecewise-constant strategy v^{str} *guarantees evasion* starting from initial state x_0 if $x^{\text{mot}}(T, x_0, u, v^{\text{str}}) \notin M$ for all admissible realization of the pursuer's control u. Based on algorithms operating with polygons, we propose the piecewise-constant strategies u^{str} and v^{str} such that for any initial state x_0 the following alternative holds: either the strategy v^{str} guarantees evasion, or the strategy u^{str} guarantees ε-pursuit starting from x_0 with $\varepsilon = O(\tau + \sigma)$, where σ is the maximal length of edges of the polygons.

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