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Construction of the Generalized Solution to a Molecular Genetic Model

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Abstract: The following Cauchy problem with state constraints for Hamilton–Jacobi equation is considered:

$$\partial u/\partial t + H(x, \partial u/\partial x) = 0, \quad t \geq 0, \quad x \in [-1; 1], \quad u(0, x) = u_0(x) \tag{1}$$

$$H(x, p) = -f(x) + 1 - \frac{1+x}{2} e^{2p} - \frac{1-x}{2} e^{-2p}, \tag{2}$$

This problem arises in molecular biology for Crow-Kimura model of evolution genetics [1]. A concept of continuous generalized solutions to the boundary problem with state constraints is suggested [2, 3]. It is proven that the solutions exist and can be defined with the help of viscosity and minimax solutions to auxiliary Dirichlet problems. Additional vital requirements on structure of solutions inside the domain are assumed. A construction of the generalized solution with additional properties is suggested. It is basing on calculus of variations methods and dynamic programming. This approach can be considered as a generalization of the classical Cauchy method of characteristics. Properties of the characteristics are studied and applied to the construction of generalized solution with prescribed structure. Simulation results are given.

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[2] *Subbotina N.N., Shagalova L.G.* On a solution to the Cauchy problem for the HamiltonJacobi equation with state constraints // Trudy Inst. Mat. i Mekh. UrO RAN. 2011. Vol. 17, no. 2, pp.191–208.[in Russian].

[3] *Subbotina N.N., Kolpakova E.A, Tokmantcev T.B., Shagalova L.G.* The Method of Characteristics for Hamilton - Jacobi - Bellman Equations. Ekaterinburg. RIO, UrB RAS. 2013. 244 p. [in Russian]