

Dynamic Programming to Reconstruction Problems for a Macroeconomic Model

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Abstract: A model of macroeconomics is considered due to works by E. G. Al'brekht [1]. The model describes production activity for single firms, various branches of industry or industry and economics of a region. In order to obtain scenarios of a short-term as well as a long-term prediction we consider problems of identification and reconstruction of controls and trajectories of the model:

$$\frac{dx_1}{dt} = u_1(t) \frac{\partial G(x_1, x_2)}{\partial x_1}, \quad \frac{dx_2}{dt} = u_2(t) \frac{\partial G(x_1, x_2)}{\partial x_2} \quad (1)$$

on a time interval $t \in [0, T]$. Here x_1 denotes the gross product, x_2 denotes production costs, $u_1(t)$, $u_2(t)$ are controlling parameters, satisfied the geometric restrictions

$$(u_1, u_2) \in U = \{|u_1| \leq U_1, \quad |u_2| \leq U_2\}, \quad (2)$$

where $U_1 > 0$, $U_2 > 0$ are constants. The function $G(x_1, x_2)$ is called the macroeconomic potential of the system. It has the form $G(x_1, x_2) = x_1 x_2 (a_0 + a_1 x_1 + a_2 x_2)$, a_0, a_1, a_2 are constants. It is assumed that the discrete sampling history and the sampling error estimate are known. A given function $y(\cdot) : [0, T] \rightarrow R^2$ is a continuous interpolation of the statistic data. Problems of identification and reconstruction are under consideration [2, 1].

Auxiliary variational problems are introduced to minimize integral functionals of the positive and negative regularized discrepancy.

It is proven that trajectories and controls of the system constructed with the help of Dynamic Programming solve the perturbed inverse problem.

- [1] *Subbotina N.N., Kolpakova E.A., Tokmantsev T.B., Shagalova L.G.* The Method of Characteristics for Hamilton - Jacobi - Bellman Equations. Ekaterinburg. RIO, UrB RAS. 2013. 244 p. [in Russian]
- [2] *Osipov Yu.S., Kryazhimskii A.V., Maksimov V.I.* Some Algorithms for the Dynamic Reconstruction of Inputs. Proceedings of the Steklov Institute of Mathematics. Springer. 2011. Vol. 275. Suppl. 1, pp. S86-S120.