27th IFIP TC7 Conference 2015 on System Modelling and Optimization

Optimality principles, dynamic programming, and applied problems

Optimal boundary control problems for steady-state models of complex heat transfer

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Abstract: Optimal control problems for models of complex heat transfer in scattering media with reflecting boundaries are of great importance in connection with engineering applications. The steady-state diffusion model (P_1 approximation) describing radiative, conductive, and convective heat transfer processes in a bounded domain $\Omega \subset \mathbb{R}^3$ is considered. The governing equations have the following normalized form:

$$-a\Delta\theta + \mathbf{v}\cdot\nabla\theta + b\kappa_a|\theta|\theta^3 = b\kappa_a\varphi, \quad -\alpha\Delta\varphi + \kappa_a\varphi = \kappa_a|\theta|\theta^3. \qquad (1)$$

Here θ is the normalized temperature, φ the normalized radiation intensity averaged over all directions, $\mathbf{v}$ a prescribed velocity field of the medium, and κ_a the absorption coefficient. The constants a, b, and α are given. The following boundary conditions on $\Gamma := \partial\Omega$ are assumed:

$$a\partial_n\theta + \beta(\theta - u_0)|_\Gamma = 0, \quad \alpha\partial_n\varphi + u_1(\varphi - u_0^4)|_\Gamma = 0, \qquad (2)$$

where β is given positive function defined on Γ, and the symbol ∂_n denotes the derivative in the outward normal direction. Boundary controls are pairs, $u = \{u_0, u_1\}$, where the first function represents the boundary temperature, and the second one defines reflection properties of the boundary. An objective functional is to be minimize.

In this talk, a priori estimates of solutions of control system (1) and (2) ensuring the solvability of the control problem and providing optimality conditions of first order are presented. A class of monotone functionals for which the optimality conditions do not require adjoint equations is considered. The notion of strong optimal control, which is optimal for all objective functionals of the above class, is introduced. The theoretical analysis is illustrated by numerical examples.