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Inversion, Estimation and Control of Uncertain Distributed Dynamical Systems

# Recent results on the controllability of the wave equation with persistent memory

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**Abstract:** We survey recent results on controllability of distributed systems with persistent memory under the action of a control acting in the Dirichlet boundary condition. We concentrate on the boundary control of a system of Gurtin-Pipkin type, i.e. a system described by

$$w'' = \Delta w + \int_0^t M(t-s)\Delta w(s)\mathrm{d}\, s\,. \qquad (\mathbf{A})$$

Here $w = w(x,t)$ where $x \in \Omega \subseteq \mathbb{R}^n$ (a region with smooth boundary). We impose

$$w(x,0) = 0\,, \quad w'(x,0) = 0\,, \qquad w = f \text{ on } \Gamma \subseteq \partial\Omega\,.$$

The *associate wave equation to* $(\mathbf{A})$ is the equation $(\mathbf{A})$ with $M = 0$.
Like in the case of the wave equation, controllability is the property that it is possible to hit a target $(\xi, \eta) \in L^2(\Omega) \times H^{-1}(\Omega)$ by using a suitable (square integrable) control. Note that, different from the wave equation, this property is not controllability in the sense of Kalman, but it extends the property of "relative controllability" in the sense of Gabasov and Kirillova. This (weaker) controllability property is the right concept for the identification of initial conditions (when they are nonzero, i.e. observability) and source reconstruction.
Controllability holds for system $(\mathbf{A})$ if $\Gamma$ and $T$ are such that the associated wave equation is controllable. This fact has been proved using different methods, under more or less general assumptions. Here we show some of the available results and in particular we combine the moment methods with the cosine operator approach in order to give a simple proof.
See [1] for references.

# References

[1] Pandolfi, L.: Distributed systems with persistent memory, SpringerBriefs in Control, Automation and Robotics, 2014.