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Optimization and Control of Nonsmooth and Complementarity-Based Systems: Theory and Numerics

On Walras-Cournot-Nash equilibria and their computation

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Abstract: The contribution is devoted to a model of oligopolistic market, where the agents/ producers share a certain amount of the co-called *rare resource*. In this equilibrium model, developed by S.D. Flåm in 2013, the rare resource is allocated to single producers by a national or international authority. Moreover, it is transferable, so that after the initial allocation has been conducted, it may be handled in the market. The optimal strategy of each producer, acting in the market, consists thus from two components: the own production and the amount of rare resource which he/she intends to sell or to purchase (in addition to his/her initial endowment). One has thus a joint price mechanism which leads eventually both to the price of the produced commodity (via the respective inverse demand function) as well as to the price of the rare resource. We speak about *Walras-Cournot-Nash* (WCN) equilibria.

In contrast to the approach of S.D. Flåm, where the price-building process is considered as a two-phase evolutionary equilibrium, we look at the problem from the perspective of the authority providing the allocation and compute the WCN equilibria *at once* by solving the appropriate variational inequality.

In this contribution we will present first a simple proof for the existence of WCN equilibria, based on the boundedness of the equilibrium price of the rare resource. Thereafter we will discuss various possible approaches to their computation and, finally, examine the local stability of WCN equilibria with respect to small changes of problem data. The achieved stability results enable, for instance, the mentioned authority to perform an optimization of the overall allocated amount of the rare resource with respect to a respective social welfare objective.