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Numerical Analysis for PDE-Constrained Optimal Control Problems

The Nitsche Trick for the Obstacle Problem – A Counterexample and Consequences for Optimal Control

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Abstract: We consider the Finite Element (FE) discretization of the obstacle problem using piecewise linear and continuous finite elements. While a priori error estimates in the energy space are standard and well known, the classical Nitsche trick for improved error estimates in $L^2(\Omega)$ seems to fail due to a lack of regularity in the dual problem. This is demonstrated by a one-dimensional counterexample, which provides a (rigorously computable) order of convergence of $2 - 1/p$, if the obstacle is described by a function in $W^{2,p}(\Omega)$. The counterexample is based on a discretization of the obstacle by means of Lagrange interpolation. The L^2-a priori estimate directly affects the convergence analysis for an optimal control problem governed by the obstacle problem.

This is a joint work with Constantin Christof (TU Dortmund).