27th IFIP TC7 Conference 2015 on System Modelling and Optimization

[Stochastic modeling, control and applications]

[On linear stochastic volatility model]

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Abstract: [I present the new results on a linear stochastic volatility model, i.e. a model of market defined on a complete probability space in which the price X_t at time t of the underlying asset has a stochastic volatility Y_t, and the dynamics of the vector (X, Y) is given by

$$dX_t = Y_t X_t dW_t, \tag{1}$$

$$dY_t = \mu(t, Y_t)dt + \sigma(t, Y_t)dZ_t, \tag{2}$$

where X_0, Y_0 are positive constants, the processes W, Z are correlated Brownian motions, $d\langle W, Z\rangle_t = \rho dt$ with $\rho \in (-1, 1)$, and $\mu : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$, $\sigma : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ are continuous functions such that there exists a unique strong solution of (2), which is positive and $\int_0^T Y_u^2 du < \infty$ $\mathbb{P}$-a.s. The process X is a local martingale, so there is no arbitrage on the market so defined. Note that the known models such as Black and Scholes model, log-normal stochastic volatility model, Heston model (where Y^2 is a CIR process) and Stein and Stein model belong to this class. It turns out that under some natural assumptions the distribution of the asset price has a density function, which has nice representation. The representations of the arbitrage prices of vanilla European options (i.e. call and put option) will be also presented. As an application of these general results I present new results for the log-normal stochastic volatility model (Hull and White model). The presentation is based on the joint work with Maciej Wiśniewolski.]